Why Portfolios?

You will recall that expected return from individual securities carries some degree of risk. *Risk* was defined as the standard deviation around the expected return.[1] In effect we equated a security's risk with the variability of its return. More dispersion or variability about a security's expected return meant the security was riskier than one with less dispersion.

The simple fact that securities carry differing degrees of expected risk leads most investors to the notion of holding more than one security at a time, in an attempt to spread risks by not putting all their eggs into one basket.[2] Diversification of one's holdings is intended to reduce risk in an economy in which every asset's returns are subject to some degree of uncertainty. Even the value of cash suffers from the inroads of inflation. Most investors hope that if they hold several assets, even if one goes bad, the others will provide some protection from an extreme loss.

Diversification

Efforts to spread and minimize risk take the form of diversification. The more traditional forms of diversification have concentrated upon holding a number of security types (stock, bonds) across industry lines (utility, mining, manufacturing groups). The reasons are related to inherent differences in bond and equity contracts, coupled with the notion that an investment in firms in dissimilar industries would most likely do better than in firms within the same industry. Holding one stock each from mining, utility, and manufacturing groups is superior to holding three mining stocks. Carried to its extreme, this approach leads to the conclusion that the best diversification comes through holding large numbers of securities scattered across industries. Many would feel that holding fifty such scattered stocks is five times more diversified than holding ten scattered stocks.

Most people would agree that a portfolio consisting of two stocks is probably less risky than one holding either stock alone. However, experts disagree with

[1]Standard deviation is a risk surrogate, not a synonym for risk. For a review of some aspects of risk, see Fred D. Arditti, "Risk and the Required Return on Equity," *Journal of Finance* (March 1967), pp. 19–36.

[2]Note that some advocate a concentration philosophy. This point of view stresses "putting all your eggs into one basket and keeping a sharp eye on the basket." See, for example, a classic. Gerald M. Loeb, *The Battle for Investment Survival* (New York: Simon and Schuster, 1965).

regard to the "right" kind of diversification and the "right" reason. The discussion that follows introduces and explores a formal, advanced notion of diversification conceived by the genius of Harry Markowitz.[3] Markowitz's approach to coming up with good portfolio possibilities has its roots in risk-return relationships. This is not at odds with traditional approaches in concept. The key differences lie in Markowitz's assumption that investor attitudes toward portfolios depend exclusively upon (1) expected return and risk, and (2) quantification of risk. And risk is, by proxy, the statistical notion of variance, or standard deviation of return. These simple assumptions are strong, and they are disputed by many traditionalists.[4]

EFFECTS OF COMBINING SECURITIES

Although holding two securities is probably less risky than holding either security alone, *is it possible to reduce the risk of a portfolio by incorporating into it a security whose risk is greater than that of any of the investments held initially?* For example, given two stocks, X and Y, with Y considerably more risky than X, a portfolio composed of some of X and some of Y may be less risky than a portfolio composed exclusively of the less risky asset, X.

Assume the following about stocks X and Y:

	STOCK X	STOCK Y
Return (%)	7 or 11	13 or 5
Probability	.5 each return	.5 each return
Expected return (%)	9*	9[†]
Variance (%)	4	16
Standard deviation (%)	2	4

*Expected return = (.5)(7) + (.5)(11) = 9
[†]Expected return = (.5)(13) + (.5)(5) = 9

Clearly, although X and Y have the same expected return, 9 percent, Y is riskier than X (standard deviation of 4 versus 2). Suppose that when X's return is high, Y's return is low, and vice versa. In other words, when the return on X is 11 percent, the return on Y is 5 percent; similarly, when the return on X is 7 percent, the return

[3]Harry M. Markowitz, *Portfolio Selection: Efficient Diversification of Investments* (New York: John Wiley, 1959). Competing portfolio models are found in Henry A. Latané, "Investment Criteria—A Three-Asset Portfolio Balance Model," *Review of Economics and Statistics* 45 (November 1963), pp. 427–30; and Jack Hirschleifer, "Investment Decision under Uncertainty: Application of the State-Preference Approach," *Quarterly Journal of Economics* 80 (May 1966), pp. 252–77.

[4]For a look at how practitioners view some aspects of the Markowitz approach, see Frank E. Block, "Elements of Portfolio Construction," *Financial Analysts Journal* (May–June 1969), pp. 123–29.

on Y is 13 percent. Question: Is a portfolio of some X and some Y in any way superior to an exclusive holding of X alone (has it less risk)?

Let us construct a portfolio consisting of two-thirds stock X and one-third stock Y. The average return of this portfolio can be thought of as the weighted-average return of each security in the portfolio; that is:

$$R_p = \sum_{i=1}^{N} X_i R_i \tag{17.1}$$

where:

R_p = expected return to portfolio
X_i = proportion of total portfolio invested in security i
R_i = expected return to security i
N = total number of securities in portfolio

Therefore,

$$R_p = (\tfrac{2}{3})(9) + (\tfrac{1}{3})(9) = 9$$

But what will be the range of fluctuation of the portfolio? In periods when X is better as an investment, we have $R_p = (\tfrac{2}{3})(11) + (\tfrac{1}{3})(5) = 9$; and similarly, when Y turns out to be more remunerative, $R_p = (\tfrac{2}{3})(7) + (\tfrac{1}{3})(13) = 9$. Thus, by putting part of the money into the riskier stock, Y, we are able to *reduce* risk considerably from what it would have been if we had confined our purchases to the less risky stock, X. If we held only stock X, our expected return would be 9 percent, which could in reality be as low as 7 percent in bad periods or as much as 11 percent in good periods. The standard deviation is equal to 2 percent. Holding a mixture of two-thirds X and one-third Y, our expected and experienced return will always be 9 percent, with a standard deviation of zero. We can hardly quarrel with achieving the same expected return for less risk. In this case we have been able to eliminate risk altogether.

The reduction of risk of a portfolio by blending into it a security whose risk is *greater than* that of any of the securities held initially suggests that deducing the riskiness of a portfolio simply by knowing the riskiness of individual securities is not possible. It is vital that we also know the *interactive risk* between securities!

The crucial point of how to achieve the proper proportions of X and Y in reducing the risk to zero will be taken up later. However, the general notion is clear. The risk of the portfolio is reduced by playing off one set of variations against another. Finding two securities each of which tends to perform well whenever the other does poorly makes more certain a reasonable return for the portfolio as a whole, even if one of its components happens to be quite risky.

This sort of hedging is possible whenever one can find two securities whose behavior is inversely related in the way stocks X and Y were in the illustration. Now we need to take a closer look at the matter of how securities may be correlated in terms of rate of return.

A Closer Look at Portfolio Risk

The risk involved in individual securities can be measured by standard deviation or variance. When two securities are combined, we need to consider their interactive risk, or *covariance.* If the rates of return of two securities move together, we say their interactive risk or covariance is positive. If rates of return are independent, covariance is zero. Inverse movement results in covariance that is negative. Mathematically, covariance is defined as

$$\text{cov}_{xy} = \frac{1}{N}\sum^{N}[R_x - \overline{R}_x][R_y - \overline{R}_y]$$

where the probabilities are equal and

cov_{xy} = covariance between x and y
R_x = return on security x
R_y = return on security y
$\overline{R}_x$ = expected return to security x
$\overline{R}_y$ = expected return to security y
N = number of observations

Using our earlier example of stocks X and Y:

	RETURN	EXPECTED RETURN	DIFFERENCE
Stock X	7	9	−2
Stock Y	13	9	4
			Product −8
Stock X	11	9	2
Stock Y	5	9	−4
			Product −8

$$\text{cov} = \frac{1}{2}[(7-9)(13-9) + (11-9)(5-9)]$$

$$= \frac{1}{2}[(-8) + (-8)] = \frac{-16}{2} = -8$$

Instead of squaring the deviations of a single variable from its mean, we take two corresponding observations of the two stocks in question at the *same point in time,* determine the variation of each from its expected value, and multiply the two deviations together. If whenever x is below its average, so is y, then for those periods each deviation will be negative, and their product consequently will be positive. Hence we will end up with a covariance made up of an average of positive values, and its value will be large. Similarly, if one of the variables is relatively large whenever the other is small, one of the deviations will be positive and the other negative, and the covariance will be negative. This is true with our example above.

The *coefficient of correlation* is another measure designed to indicate the similarity or dissimilarity in the behavior of two variables. We define

$$r_{xy} = \frac{\text{cov}_{xy}}{\sigma_x \sigma_y}$$

where:

r_{xy} = coefficient of correlation of x and y
cov_{xy} = covariance between x and y
σ_x = standard deviation of x
σ_y = standard deviation of y

The coefficient of correlation is, essentially, the covariance taken not as an absolute value but relative to the standard deviations of the individual securities (variables). It indicates, in effect, how much x and y vary together as a proportion of their combined individual variations, measured by $\sigma_x \sigma_y$. In our example, the coefficient of correlation is

$$r_{xy} = -8/[(2)(4)] = -8/8 = -1.0$$

If the coefficient of correlation between two securities is -1.0, then a perfect negative correlation exists (r_{xy} cannot be less than -1.0). If the correlation coefficient is zero, then returns are said to be independent of one another. If the returns on two securities are perfectly correlated, the correlation coefficient will be $+1.0$, and perfect positive correlation is said to exist (r_{xy} cannot exceed $+1.0$).

Thus, correlation between two securities depends upon (1) the covariance between the two securities, and (2) the standard deviation of each security.

Portfolio Effect in the Two-Security Case

We have shown the effect of diversification on reducing risk. The key was not that two stocks provided twice as much diversification as one, but that by investing in securities with negative or low covariance among themselves, we could reduce the risk. Markowitz's efficient diversification involves combining securities with less than positive correlation in order to reduce risk in the portfolio without sacrificing any of the portfolio's return. In general, the lower the correlation of securities in the portfolio, the less risky the portfolio will be. This is true regardless of how risky the stocks of the portfolio are when analyzed in isolation. It is not enough to invest in *many* securities; it is necessary to have the *right* securities.

Let us conclude our two-security example in order to make some valid generalizations. Then we can see what three-security and larger portfolios might be like. In considering a two-security portfolio, portfolio risk can be defined more formally now as:

$$\sigma_p = \sqrt{X_x^2\sigma_x^2 + X_y^2\sigma_y^2 + 2X_xX_y(r_{xy}\sigma_x\sigma_y)} \qquad (17.2)$$

where:

σ_p = portfolio standard deviation
X_x = percentage of total portfolio value in stock X
X_y = percentage of total portfolio value in stock Y
σ_x = standard deviation of stock X
σ_y = standard deviation of stock Y
r_{xy} = correlation coefficient of X and Y

Note: $r_{xy}\sigma_x\sigma_y = \text{cov}_{xy}$.

Thus, we now have the standard deviation of a portfolio of two securities. We are able to see that portfolio risk (σ_p is sensitive to (1) the proportions of funds devoted to each stock, (2) the standard deviation of each stock, and (3) the covariance between the two stocks. If the stocks are independent of each other, the correlation coefficient is zero ($r_{xy} = 0$). In this case, the last term in Equation 17.2 is zero. Second, if r_{xy} is greater than zero, the standard deviation of the portfolio is greater than if $r_{xy} = 0$. Third, if r_{xy} is less than zero, the covariance term is negative, and portfolio standard deviation is less than it would be if r_{xy} were greater than or equal to zero. Risk can be totally eliminated only if the third term is equal to the sum of the first two terms. This occurs only if first, $r_{xy} = -1.0$, and second, the percentage of the portfolio in stock X is set equal to $X_x = \sigma_y/(\sigma_x + \sigma_y)$.

To clarify these general statements, let us return to our earlier example of stocks X and Y. In our example, remember that:

	STOCK X	STOCK Y
Expected return (%)	9	9
Standard deviation (%)	2	4

We calculated the covariance between the two stocks and found it to be −8. The coefficient of correlation was −1.0. The two securities were perfectly negatively correlated.

CHANGING PROPORTIONS OF X AND Y

What happens to portfolio risk as we change the total portfolio value invested in X and Y? Using Equation 17.2, we get:

STOCK X (%)	STOCK Y (%)	PORTFOLIO STANDARD DEVIATION
100	0	2.0
80	20	0.8
66	34	0.0
20	80	2.8
0	100	4.0

Notice that portfolio risk can be brought down to zero by the skillful balancing of the proportions of the portfolio to each security. The preconditions were $r_{xy} = -1.0$ and $X_x = \sigma_y/(\sigma_x + \sigma_y)$, or $4/(2 + 4) = .666$.

Changing the Coefficient of Correlation

What would be the effect using $x = ⅔$ and $y = ⅓$ if the correlation coefficient between stocks X and Y had been other than −1.0? Using Equation 17.2 and various values for r_{xy}, we have:

r_{xy}	PORTFOLIO STANDARD DEVIATION
−0.5	1.34*
0.0	1.9
+0.5	2.3
+1.0	2.658

$$^*\sigma_p = \sqrt{(.666)^2(2)^2 + (.334)^2 + (2)(.666)(.334)(-.5)(2)(4)}$$
$$= \sqrt{1.777 + 1.777 - (.444)(4)} = \sqrt{1.777} = 1.34$$

If no diversification effect had occurred, then the total risk of the two securities would have been the weighted sum of their individual standard deviations:

$$\text{Total undiversified risk} = (.666)(2) + (.334)(4) = 2.658$$

Because the undiversified risk is equal to the portfolio risk of perfectly positively correlated securities ($r_{xy} = +1.0$), we can see that favorable portfolio effects occur only when securities are not perfectly positively correlated. The risk in a portfolio is less than the sum of the risks of the individual securities taken separately whenever the returns of the individual securities are not perfectly positively correlated; also, the smaller the correlation between the securities, the greater the benefits of diversification. A negative correlation would be even better.

In general, some combination of two stocks (portfolios) will provide a smaller standard deviation of return than either security taken alone, as long as the correlation coefficient is less than the ratio of the smaller standard deviation to the larger standard deviation:

$$r_{xy} < \frac{\sigma_x}{\sigma_y}$$

Using the two stocks in our example:

$$-1.00 < \frac{2}{4}$$

$$-1.00 < +.50$$

If the two stocks had the same standard deviations as before but a coefficient of correlation of, for example, +.70, there would have been no portfolio effect because +.70 is not less than +.50.

GRAPHIC ILLUSTRATION OF PORTFOLIO EFFECTS

The various cases where the correlation between two securities ranges from −1.0 to +1.0 are shown in Figure 17–1. Return is shown on the vertical axis and risk is measured on the horizontal axis. Points *A* and *B* represent pure holdings (100 percent) of securities A and B. The intermediate points along the line segment *AB* represent portfolios containing various combinations of the two securities. The line segment identified as $r_{ab} = +1.0$ is a straight line. This line shows the inability of a portfolio of perfectly positively correlated securities to serve as a means to reduce variability or risk. Point *A* along this line segment has no points to its left; that is, there is no portfolio composed of a mix of our perfectly correlated securities A and B that has a lower standard deviation than the standard deviation of A. Neither A nor B can help offset the risk of the other. The wise investor who wished to minimize risk would put all his eggs into the safer basket, stock A.

The segment labeled $r_{ab} = 0$ is a hyperbola. Its leftmost point will not reach the vertical axis. There is no portfolio where $\sigma_p = 0$. There is, however, an inflection just above point *A* that we shall explain in a moment.

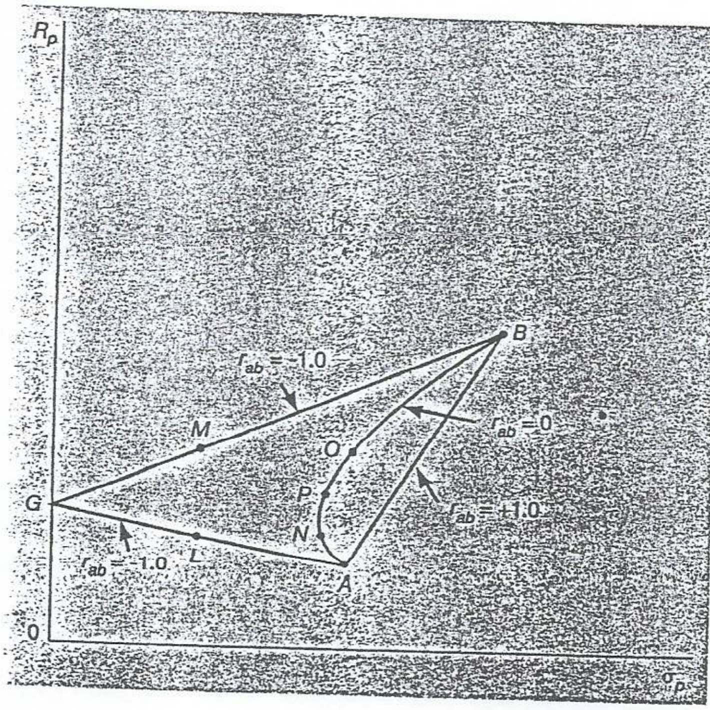

FIGURE 17–1 Portfolios of Two Securities with Differing Correlation of Returns

The line segment labeled $r_{ab} = -1.0$ is compatible with the numerical example we have been using. This line shows that with perfect inverse correlation, portfolio risk can be reduced to zero. Notice points L and M along the line segment AGB, or $r_{ab} = -1.0$. Point M provides a higher return than point L, while both have equal risk. Portfolio L is clearly inferior to portfolio M. All portfolios along the segment GLA are clearly inferior to portfolios along the segment GMB. Similarly, along the line segment APB, or $r_{ab} = 0$, segment BOP contains portfolios that are superior to those along segment PNA.

Markowitz would say that all portfolios along all line segments are "*feasible*," but some are more "*efficient*" than others.